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INFLUX OF OIL TO A GALLERY OF WATER-PLUGGED WELLS

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Within the framework of the Buckley–Leverett scheme, a solution is obtained for the problem of organization of an influx to a gallery of wells whose near-well zone is contaminated for some reasons by the water phase. A method of engineering estimates is developed for the moment of penetration of the front of water displacing oil into the plugged zone of production wells with simultaneous determination of oil recovery in the reservoir. The results obtained may be used in constructing a mathematical model of optimized development of oil fields.

Introduction. The Buckley–Leverett mathematical model of immiscible displacement is the simplest one in the theory of filtration of two-phase fluids. The assumption on the absence of the capillary jump of pressure at the boundary of movable incompressible liquid phases, which is used in this model, allows one to simplify the initial system of differential equations and decrease its order by one. Nevertheless, even in the case of one-dimensional motion with an arbitrarily prescribed initial oil saturation in the reservoir, the solution of initial-boundary problems implies overcoming of technical difficulties [1, p. 362–398] caused by the formation and propagation of saturation discontinuities whose determination requires the use of integral laws of conservation of mass.

Within the Buckley–Leverett model, we consider the problem of double displacement: organization of an influx to the gallery (chain) of productive wells partly plugged by water under the action of excess pressure in a parallel gallery of wells pumping water into the reservoir. Partial plugging can be caused by penetration of the drilling agent into the reservoir, stoppage of wells by plugging solutions, or hydraulic fracture of the reservoir. In the case of the influx, part of the filtrate remains in the near-well zone; as a result, its permeability for the oil phase is lower than in the main reservoir. The moment when oil-displacing water reaches the partly plugged near-well zone is important in increasing oil production. From this moment, intense water encroachment of the well begins. Access of oil into the productive well is hindered by the increasing effect of capillary forces [2, 3], which can be taken into account using a more complicated model of oil displacement.

1. Formulation of the Problem. We consider a reservoir of unit length. There is a gallery of production wells in the cross section $x = 0$ and a gallery of injection wells in the cross section $x = 1$. The motion is assumed to be one-dimensional and directed opposite to the x axis. In accordance with the Buckley–Leverett approach, the pressure $p(x,t)$ in both incompressible fluids is identical. Without loss of generality, we may assume that $p(0,t) = 0$ and $p(1,t) = \Delta p(t)$, where the pressure difference $\Delta p(t)$ is an arbitrary function of the time t.

Let at the initial time $t = 0$ the near-well zone of production wells be "contaminated" by water infiltrate at a distance $0 \leqslant x \leqslant x_0 = \text{const}$. We denote the oil saturation by s ($1 - s$ is the water saturation). For $t = 0$ and $0 \leqslant x \leqslant x_0$, we have the oil saturation $s(0,x) < 1$. To simplify calculations, we assume that $s(0,x) = s_0 = \text{const}$ in this region, where $s_0 < 1$ is the mean oil saturation chosen on the basis of the ratio of viscosities $\alpha = \mu_1/\mu$ of the immiscible phases (μ and μ_1 are the viscosities of oil and water, respectively).

After the beginning of motion in the reservoir, four regions changing with time should be distinguished: 1) region of uniform two-phase motion ($x \in [0, x_1(t)]$); 2) region of two-phase motion ($x \in [x_1(t), x_0]$); 3) region of filtration of the homogeneous fluid (oil) ($x \in [x_0, x_2(t)]$), $s \equiv 1$; 4) region of two-phase motion ($x \in [x_2(t), 1]$). Here $x_1(t)$ and $x_2(t)$ are the fronts of oil and water motion, where jumplike changes in saturation occur.

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In regions 1–3, the initial system of equations is formed by the generalized Darcy laws for both phases

$$v = -\frac{K}{\mu} f(s) \frac{\partial p}{\partial x}, \qquad v_1 = -\frac{K}{\mu_1} f_1(s) \frac{\partial p}{\partial x} \tag{1}$$

and the laws of conservation of masses

$$m \frac{\partial s}{\partial t} + \frac{\partial v}{\partial x} = 0, \qquad -m \frac{\partial s}{\partial t} + \frac{\partial v_1}{\partial x} = 0, \tag{2}$$

where m is the porosity, K is the permeability of the medium, v, v_1 and f, f_1 are the velocities and relative phase permeabilities for oil and water, respectively.

Following [4, p. 113], we assume that $f(s) = s^n$, where $n = 3.3$ (according to Slichter) and $n = 4$ (according to Kozeny). Since capillary forces are ignored and permeability is a characteristic of the effective porous space of the medium, we should also assume that $f_1(s) = f(1-s) = (1-s)^n$. It is known that system (1), (2) has the first integral $v + v_1 = V(t)$ [5]. The fluid velocities are determined by the formulas

$$v = V(t)F(s), \qquad v_1 = V(t)(1 - F(s)),$$

where $F(s) = \alpha f/[\alpha f + f(1-s)]$ is a function that possesses the properties $F(0) = 0$, $F(1) = 1$, and $F'(0) = F'(1) = 0$ and has one maximum at the point $s = s_*$. The value of $s = s_*$ is found from the transcendental equation $(1 - s_*)[(n-1)/2 + s_*] = \alpha s_*^n[(n+1)/2 - s_*]$. The overall velocity $V(t)$ and the front positions $x_1(t)$ and $x_2(t)$ are to be determined in the course of solving the problem.

Note that, in region 1 of uniform two–phase motion, where $s \equiv s_0 = \text{const}$ for $t = 0$, the relations $v = v_0 = V(t)F(s_0)$ and $v_1 = v_{1,0} = V(t)(1 - F(s_0))$ are valid, and the prescribed constant saturation is also preserved for $t > 0$. In region 3, we have $v = V(t)$ and $v_1 = 0$ ($s \equiv 1$); therefore, the pressure distribution is a linear function of the coordinate in accordance with the conventional Darcy law of motion.

To describe the motion of the phases in regions 2 and 4, system (1), (2) is reduced to one first-order equation

$$\frac{\partial s}{\partial \tau} + F'(s) \frac{\partial s}{\partial x} = 0, \tag{3}$$

where $\tau = \dfrac{1}{m}\displaystyle\int_0^t V(t)\,dt < 0$ is the total volume of phases, which is "pumped" during the time $t > 0$ in the negative direction of the x axis through the whole reservoir (as a consequence of incompressibility of the fluids).

The necessity of introducing the fronts $x_1(t)$ and $x_2(t)$ of saturation discontinuity becomes obvious if we rewrite Eq. (3) in Lagrangian coordinates, the new sought function being the x coordinate of the particle that has the saturation s at the "time" τ, i.e., $x = X(s, \tau)$ and $x^0 = X(s, 0)$ is the initial position of the particle. By calculating the derivatives by the formulas

$$\frac{\partial s}{\partial x} = \Big(\frac{\partial X}{\partial s}\Big)^{-1}, \qquad \frac{\partial s}{\partial \tau} = \frac{\partial X}{\partial \tau}\Big(\frac{\partial X}{\partial s}\Big)^{-1},$$

we convert Eq. (3) to the form

$$\frac{\partial X}{\partial \tau} = F'(s). \tag{4}$$

2. Construction of the Solution. Equation (4) has the known general integral [5]

$$X(s, \tau) = \tau F'(s) + x^0(s), \tag{5}$$

which is obtained from Eq. (3) by the method of characteristics. As applied to regions 2 and 4, the solutions (5) acquire the following form:

$$X(s, \tau) = \tau F'(s) + x^0(s) \qquad [\tau = 0\colon \quad s(x_0 - 0) = s_0, \quad s(x_0 + 0) = 1]; \tag{6}$$

$$X(s, \tau) = \tau F'(s) + 1 \qquad [\tau = 0\colon \quad s(1 - 0) = 1, \quad s(1 + 0) = 0]. \tag{7}$$

Since the function $F'(s)$ has an ascending [from 0 to $F'(s_*)$] and descending [from $F'(s_*)$ to 0] branches, the use of Eqs. (6) and (7) within the entire range of variation of the independent variable s in initial-boundary data becomes impossible if this interval contains the point $s = s_*$. In the latter case, one should match solutions determined uniquely on two nonintersecting intervals of the variable s, one belonging to $0 < s < s_*$ and the other

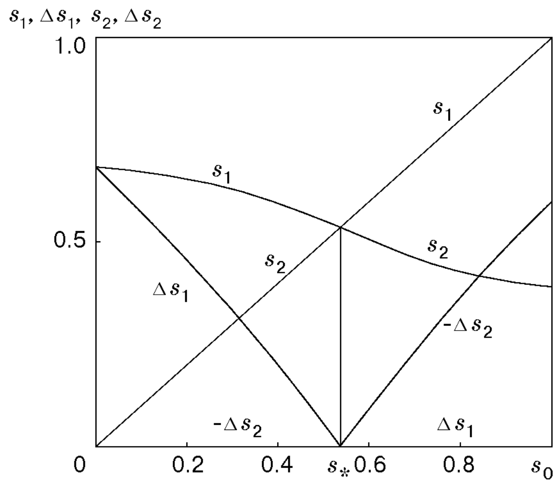

Fig. 1

to $s_* < s < 1$. This matching leads to saturation discontinuities at the left boundaries of regions 2 and 4: $x = x_1(t)$ and $x = x_2(t)$, respectively. The "volume velocity" of propagation of these boundaries can be found from Eq. (4)

$$\frac{\partial x_1}{\partial \tau} = F'(s_1), \qquad \frac{\partial x_2}{\partial \tau} = F'(s_2) \tag{8}$$

if the values of saturation $s = s_1$ and $s = s_2$ are defined as the boundary points of the intervals of uniqueness of solutions (6) and (7). To find the boundary points, we use the integral law of conservation of masses [5]. Thus, for region 2, this law is expressed as

$$-\int\limits_0^t [1 - F(s_0)]V(t)\,dt = m \int\limits_{x_1(t)}^{x_0} (s - s_0)\,dx.$$

After simple transformations, taking into account the representation of solution (6), the expression for τ, and the substitution of the integration variable in the integral of the right part, and integrating by parts, we obtain the transcendental equation

$$s_1 = s_0 + (F(s_1) - F(s_0))/F'(s_1), \tag{9}$$

which uniquely determines the value of $s_1 = \text{const}$ for a given s_0.

The calculations show that, for the kerosene–water system ($\alpha = 0.67$), the saturation $s_1(s_0)$ at the boundary $x_1(\tau)$ of region 2 decreases monotonically from $s_1 = 0.685$ to $s_1 = s_*$ with increasing initial saturation s_0 from 0 to $s_0 = s_* = 0.532$. The value of the saturation discontinuity $\Delta s_1(s_0) = s_1 - s_0$ decreases even stronger from $\Delta s_1(0) = 0.685$ to $\Delta s_1(s_*) = 0$. Obviously, all discontinuities in initial-boundary data ($s|_{x=x_0-0} = s_0$ and $s|_{x=x_0+0} = 1$) are smoothed from the beginning of motion in accordance with Eq. (6) for all $s_0 \geqslant s_*$, since in this case all values $s \in [s_0, 1]$ are within the region of definition of the descending branch of the function $F'(s)$. The front of the "labeled" particles moves with the "volume velocity" determined by the first formula in (8), where $s_1 = s_0$ should be set. Hence, for all $s_0 \geqslant s_*$, we obtain $\Delta s_1(s_0) \equiv 0$.

We can show that a formula similar to (9) is valid for the saturation s_2 at the moving boundary of region 4:

$$s_2 = s_0 + (F(s_2) - F(s_0))/F'(s_2).$$

Here $s_0 \leqslant 1$ is the oil saturation of the reservoir prior to water injection. In this case, the saturation s_2 increases monotonically from $s_2 = 0.381$ ($\alpha = 0.67$) to $s_2 = s_* = 0.532$ with decreasing initial saturation s_0 from 1 to $s_0 = s_*$, and the saturation discontinuity $\Delta s_2(s_0) = s_2 - s_0$ decreases from $\Delta s_2(1) = 0.381$ to $\Delta s_2(s_*) = 0$. For all $s_0 \leqslant s_*$, the identities $s_2 \equiv s_0$ and $\Delta s_2(s_0) \equiv 0$ are valid. Hence, the discontinuity in initial-boundary data ($s|_{x=1-0} = s_0$ and $s|_{x=1+0} = 0$) is naturally smoothed, since all values $s \in [0, s_0]$ are within the region of definition of the ascending branch of the function $F'(s)$. The dependences $s_1(s_0)$, $\Delta s_1(s_0)$, $s_2(s_0)$, and $\Delta s_2(s_0)$ are plotted in Fig. 1.

If the reservoir is "contaminated" by water infiltrate of the drilling fluid or plugging solutions, the motion is performed along the x axis with a velocity V_0, and the value of $\tau_0 = \dfrac{1}{m}\displaystyle\int\limits_0^t V_0(t)\,dt$ is positive. The distribution of

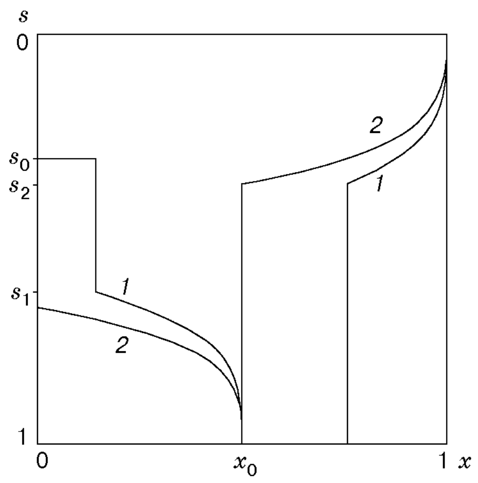

Fig. 2

saturation is determined by a formula similar to Eq. (7): $X(s, \tau_0) = \tau_0 F'(s)$ [$\tau_0 = 0$: $s(-0) = 1$, $s(+0) = 0$]. Thus, the volume of water τ_0 that penetrated into the reservoir being known, the length of the plugged zone x_0 takes the form

$$x_0 = \tau_0 F'(s_2). \tag{10}$$

We determine the mean-integral value $\langle s \rangle$ of residual oil saturation in the plugged zone by the formula

$$\langle s \rangle = \frac{1}{x_0} \int_0^{x_0} s\,dx$$

and write the equation of mass balance for water that entered the reservoir:

$$m\tau_0 = mx_0 - m \int_0^{x_0} s\,dx.$$

It follows from this equation and (10) that the value of $\langle s \rangle$ depends only on the ratio α of viscosities of the immiscible fluids and is determined by the formula

$$\langle s \rangle = 1 - 1/F'(s_2). \tag{11}$$

For example, we obtain $\langle s \rangle = 0.30$ for $\alpha = 0.67$ (kerosene–water) and $\langle s \rangle = 0.42$ for $\alpha = 0.16$. With increasing viscosity, the mean-integral volume of undisplaced oil always increases (oil recovery decreases).

Returning to constructing the solution of the problem of influx organization, we assume that the initial oil saturation s_0 of the plugged near-well zone equals the mean-integral value $\langle s \rangle$ determined by Eq. (11). The values of saturation $s_1(s_0)$ and $s_2(s_0)$ (Fig. 1) establish the ranges of the variable s in formulas (6) and (7): $s \in [s_1(s), 1]$ and $s \in [0, s_2(1)]$, respectively.

Figure 2 shows the distributions of oil saturation in the reservoir ($\alpha = 0.67$) for two values of the "pumped" volume of water: $\tau_1 = -0.17$ and $\tau_2 = -0.35$ (curves 1 and 2, respectively). The value of τ_2 is chosen from the condition $x_2 = x_0 = 0.5$, where the front of the pumped-in water merges with the front of the remaining water in the plugged zone. In our case, $\tau_2 = (1 - x_0)/F'(s_2)$. The experiments of [6] show that rapid water encroachment of production wells starts from this moment, and capillary forces hindering oil influx start to play an important role in the course of oil displacement.

To find the relationship between the variable τ and physical time t, we note that the total prescribed pressure difference $\Delta p(t)$ in the galleries is, obviously, represented as a sum of pressure differences in each region:

$$\Delta p(t) = (\Delta p(t))_1 + (\Delta p(t))_2 + (\Delta p(t))_3 + (\Delta p(t))_4. \tag{12}$$

The differences $(\Delta p(t))_1$ and $(\Delta p(t))_3$ in regions of uniform two-phase flow and uniform flow with a linear pressure distribution have the form

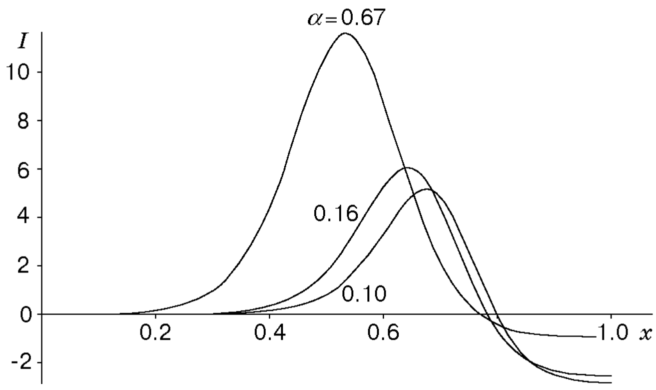

Fig. 3

$$(\Delta p)_1 = -\frac{\mu_1}{K}\,V(t)\,\frac{x_1}{\alpha f(s_0) + f(1-s_0)} = -\frac{\mu_1}{K}\,V(t)\,\frac{\tau F'(s_1) + x_0}{\alpha f(s_0) + f(1-s_0)},$$

$$(\Delta p)_3 = -\frac{\mu}{K}\,V(t)(x_2 - x_0) = -\frac{\mu}{K}\,V(t)[(\tau F'(s_2) + (1 - x_0)].$$

In regions 2 and 4, we obtain from Eq. (1)

$$\frac{\partial p}{\partial x} = -\frac{\mu}{K}\,V(t)\,\frac{F(s)}{f(s)}.$$

Therefore, the pressure differences are determined as follows:

$$(\Delta p)_2 = \int_{x_1}^{x_0} \frac{\partial p}{\partial x}\,dx = -\frac{\mu}{K}\,V(t)\tau \int_{s_1}^{1} \frac{F(s)}{f(s)}\,F''(s)\,ds,$$

$$(\Delta p)_4 = \int_{x_2}^{1} \frac{\partial p}{\partial x}\,dx = -\frac{\mu}{K}\,V(t)\tau \int_{s_2}^{0} \frac{F(s)}{f(s)}\,F''(s)\,ds.$$

We denote

$$I(x) = \int_0^x \frac{F(s)}{f(s)}\,F''(s)\,ds.$$

Figure 3 shows the functions $I(x)$ for various values of α. Substituting the resultant values of pressure differences into Eq. (12), after simple transformations, we obtain a differential equation for the variable $\tau(t)$:

$$a\,\frac{d\tau^2}{dt} - b\,\frac{d\tau}{dt} = \frac{K}{m\mu}\,\Delta p(t).$$

Here $-a/2 = \alpha F'(s_1)x_1/(\alpha f(s_0)+f(1-s_0))+F'(s_2)+I(1)-I(s_1)-I(s_2)$ and $b = \alpha x_0/(\alpha f(s_0)+f(1-s_0))+1-x_0$. Integrating the last equation with respect to t with the initial condition $\tau(0) = 0$, we obtain the quadratic equation

$$a\tau^2 - b\tau - P(t) = 0 \qquad \Big(P(t) = \frac{K}{m\mu}\int_0^t \Delta p(t)\,dt\Big),$$

which has two real roots with different signs. The negative root $\tau = (b - \sqrt{b^2 + 4aP(t)}\,)/(2a)$ has a physical meaning; this root establishes the relationship between the total linear volume of water "pumped" into the reservoir τ and the integral pressure difference $P(t)$ between the galleries of wells. As an example, we give the values of the main parameters of the problem calculated ($x_0 = 0.5$) for two values of α. We have $s_0 = 0.30$, $s_1 = 0.63$, $s_2 = 0.38$, $a = 8.58$, and $b = 1.63$ for $\alpha = 0.67$ and $s_0 = 0.42$, $s_1 = 0.73$, $s_2 = 0.50$, $a = 5.44$, and $b = 1.01$ for $\alpha = 0.16$.

Conclusions. The Buckley–Leverett system simulating the joint filtration of immiscible incompressible fluids allows one to describe the process of organization of an influx to the gallery of wells in the case of oil displacement by water. The description is adequate until water pumped into the reservoir penetrates into the near-well zone of production wells, which is partly plugged by water infiltrate, and forms a kind of a “water pipe” from one gallery to the other. At this stage of displacement, the mean-integral value of displaced oil and, hence, oil saturation are independent of the depth of water penetration and are determined by the ratio of water and oil viscosities only. At the subsequent stage of rapid water encroachment of the wells with comparatively slow washing out of the remaining oil, in our opinion, capillary forces, which are ignored in the Buckley–Leverett equations, should play an important role. The construction of the solution shows that the assumptions on the dependences of the relative permeabilities of the phases on saturation, which were accepted in [4], are not principal constraints.

The formulas obtained may be used in processing of experimental data under conditions of a standard laboratory experiment.

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